

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2015 series

4037 ADDITIONAL MATHEMATICS

4037/13

Paper 1, maximum raw mark 80

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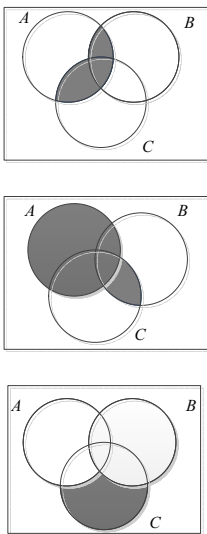
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Abbreviations

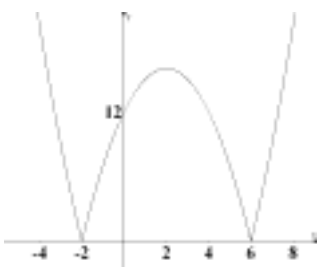
Awr	answers which round to
Cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

<p>1</p> <p>(i)</p> <p>(ii)</p> <p>(iii)</p>		<p>B1</p> <p>B1</p> <p>B1</p>	
<p>2</p>	$\cos\left(3x - \frac{\pi}{4}\right) = (\pm)\frac{1}{\sqrt{2}} \text{ oe}$ $3x - \frac{\pi}{4} = -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$ $x = \left(-\frac{\pi}{4} + \frac{\pi}{4}\right) \div 3, \left(\frac{\pi}{4} + \frac{\pi}{4}\right) \div 3, \left(\frac{3\pi}{4} + \frac{\pi}{4}\right) \div 3 \text{ oe}$ $x = 0 \text{ and } \frac{\pi}{6} \text{ (or 0 and 0.524)}$ $x = \frac{\pi}{3} \text{ (or 1.05)}$	<p>M1</p> <p>DM1</p> <p>A2/1/0</p>	<p>division by 2 and square root</p> <p>correct order of operations in order to obtain a solution</p> <p>A2 for 3 solutions and no extras in the range</p> <p>A1 for 2 solutions</p> <p>A0 for one solution or no solutions</p>

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<p>3 (a)</p> $\begin{pmatrix} 12 & 16 & 4 \\ 30 & 32 & 10 \end{pmatrix}$ <p>(b)</p> $\begin{pmatrix} 28 & -24 \\ -8 & 76 \end{pmatrix} = m \begin{pmatrix} 4 & 6 \\ 2 & -8 \end{pmatrix} + n \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ <p>$-24 = 6m$ or $-8 = 2m$ giving $m = -4$</p> <p>$28 = 4m + n$ or $76 = -8m + n$ $n = 44$</p> <p>(c)</p> $a^2 - 6 = 0$ <p>so $a = \pm\sqrt{6}$</p>		<p>B2,1,0</p> <p>B2,1,0</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B2,1,0</p>	<p>B2 for 6 elements correct, B1 for 5 elements correct</p> <p>B2 for 4 correct elements in \mathbf{X}^2 B1 for 3 correct elements in \mathbf{X}^2</p> <p>For $m = -4$ using correct I</p> <p>complete method to obtain n</p> <p>B2 for $a = \pm\sqrt{6}$ or $a = \pm 2.45$, with no incorrect statements seen or B1 for $a = \pm\sqrt{6}$ or $a = \pm 2.45$ seen or B1 for $a = \sqrt{6}$ and no incorrect working</p>
<p>4 (i)</p> $\frac{1}{2}(4\sqrt{3}+1) \times BC = \frac{47}{2}$ $BC = \frac{47}{(4\sqrt{3}+1)} \times \frac{(4\sqrt{3}-1)}{(4\sqrt{3}-1)}$ $BC = 4\sqrt{3}-1$ <p>Alternative method</p> $\frac{1}{2}(4\sqrt{3}+1) \times BC = \frac{47}{2}$ $(4\sqrt{3}+1)(a\sqrt{3}+b) = 47$ <p>Leading to $12a + b = 47$ and $a + 4b = 0$ Solution of simultaneous equations</p> $BC = 4\sqrt{3}-1$ <p>(ii)</p> $(4\sqrt{3}+1)^2 + (4\sqrt{3}-1)^2$ $= (48 + 8\sqrt{3} + 1) + (48 - 8\sqrt{3} + 1)$ $AC^2 = 98$ $AC = 7\sqrt{2} \text{ or } p = 7$		<p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1FT</p> <p>B1cao</p>	<p>correct use of the area</p> <p>correct rationalisation</p> <p>Dependent on all method being seen</p> <p>Dependent on all method seen including solution of simultaneous equations</p> <p>6 correct FT terms seen</p> <p>98 and $7\sqrt{2}$ or 98 and $p = 7$</p>

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5	<p>When $x = \frac{\pi}{4}$, $y = 2$</p> $\frac{dy}{dx} = 5\sec^2 x$ <p>When $x = \frac{\pi}{4}$, $\frac{dy}{dx} = 10$</p> <p>Equation of normal $y - 2 = -\frac{1}{10}\left(x - \frac{\pi}{4}\right)$</p> $10y + x - 20 - \frac{\pi}{4} = 0 \quad \text{or} \quad 10y + x - 20.8 = 0 \quad \text{oe}$	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>$y = 2$</p> <p>$5\sec^2 x$</p> <p>10 from differentiation</p> $y - \text{their } 2 = -\frac{1}{\text{their } 10}\left(x - \frac{\pi}{4}\right)$ <p>allow unsimplified</p>
6	<p>(i)</p>  <p>(ii)</p> <p>(2, 16)</p> <p>(iii)</p> <p>$k = 0$</p> <p>$k > 16$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p>	<p>shape</p> <p>intercepts on x-axis</p> <p>intercept on y-axis for a curve with a maximum and two arms</p> <p>$(2, \pm 16)$ seen or $(2, k)$ where $k > 0$</p> <p>$(2, 16)$ or $x = 2$ and $y = 16$ only</p>

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7	$\frac{dy}{dx} = 2 \sin 3x \quad (+c)$ $4\sqrt{3} = 2 \frac{\sqrt{3}}{2} + c$ $\frac{dy}{dx} = 2 \sin 3x + 3\sqrt{3}$ $y = -\frac{2}{3} \cos 3x + 3\sqrt{3}x \quad (+d)$ $-\frac{1}{3} = -\frac{2}{3} \cos \frac{\pi}{3} + 3\sqrt{3} \left(\frac{\pi}{9} \right) + d$ $y = -\frac{2}{3} \cos 3x + 3\sqrt{3}x - \frac{\sqrt{3}}{3} \pi$	<p>B1</p> <p>M1</p> <p>A1</p> <p>B1FT</p> <p>M1</p> <p>A1</p>	<p>2 sin 3x</p> <p>finding constant using $\frac{dy}{dx} = k \sin 3x + c$ making use of $\frac{dy}{dx} = 4\sqrt{3}$ and $x = \frac{\pi}{9}$</p> <p>Allow with $c = 5.20$ or $\sqrt{27}$</p> <p>FT integration of <i>their</i> $k \sin 3x$</p> <p>finding constant d for $k \cos 3x + cx + d$</p> <p>Allow $y = -0.667 \cos 3x + 5.20x - 0.577\pi$ or better</p>
8	<p>(a)</p> $(2 + kx)^8 = 256 + 1024kx + 1792k^2x^2 + 1792k^3x^3$ $k = \frac{1}{4}$ $p = 112$ $q = 28$ <p>(b)</p> ${}^9C_3 x^6 \left(-\frac{2}{x^2} \right)^3$ $84x^6 \left(-\frac{8}{x^6} \right) \text{ leading to } -672$	<p>B1</p> <p>B1FT</p> <p>B1FT</p> <p>M1</p> <p>DM1</p> <p>A1</p>	<p>FT 1792 multiplied by <i>their</i> k^2</p> <p>FT 1792 multiplied by <i>their</i> k^3</p> <p>correct term seen</p> <p>Term selected and 2^3 and 9C_3 correctly evaluated</p>

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9	(a) (i)	Number of arrangements with Maths books as one item = $4!$ or $4 \times 3!$	M1	$4!(\times 2)$ or $4 \times 3!(\times 2)$ oe
		or Maths books can be arranged 2! ways and History 3! ways = $2! \times 3!$		$2! \times 3!(\times 4)$ or $2 \times 3!(\times 4)$ oe
		$2 \times 4!$ or $2 \times 4 \times 3!$ or $4 \times 2 \times 3! = 48$	A1	A1 for 48
	(ii)	$5! - 48$ or $6 \times 2 \times 3!$	M1	$5!$ – <i>their</i> answer to (i)
		72	A1	or for $6 \times 2 \times 3$
	(b) (i)	3003	B1	
	(ii)	$3003 - 6 - 135$	M1	<i>their</i> answer to (i) – $6 - {}^6C_4 \times 9$
		2862	B1	135 subtracted
		2862	A1	
		or		
	$2M\ 3W = 720$	M1	complete correct method using 4 cases, may be implied by working. Must have at least one correct	
	$3M\ 2W = 1260$			
	$4M\ 1W = 756$			
	$5M = 126$	B1	any 3 correct	
	2862	A1		

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10	(i)	$10^2 = 6^2 + 6^2 - 2 \times 6 \times 6 \times \cos ABC$ or $\sin\left(\frac{ABC}{2}\right) = \frac{5}{6}$ or $ABC = \pi - \sin^{-1} \frac{10\sqrt{11}}{36}$ $ABC = 1.9702$	M1	correct cosine rule statement or correct statement for $\sin \frac{ABC}{2}$ or equating areas oe
	(ii)	$XY = 2$ Arc length $6\left(\frac{\pi - 1.970}{2}\right)$ oe Perimeter = $2 + 2\left(6\left(\frac{\pi - 1.970}{2}\right)\right)$ = 9.03	B1 B1 M1 A1	for XY (may be implied by later work, allow on diagram) correct arc length (unsimplified) <i>their</i> $2 + 2 \times 6 \times$ <i>their</i> angle C
	(iii)	$\left(\frac{1}{2} \times 6^2 \left(\frac{\pi - 1.970}{2}\right) - \frac{1}{2} \times 5 \times \sqrt{11}\right) \times 2$ = 4.50 or 4.51 or better	M1 M1 A1	sector area using <i>their</i> C area of $\triangle ABM$ where M is the midpoint of AC , or ($\triangle s$ ABY and BXY) or $\triangle ABC$ Answers to 3sf or better

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11	$x^2 - 2x - 3 = 0$ or $y^2 - 6y + 5 = 0$	M1	substitution and simplification to obtain a three term quadratic equation in one variable
	leading to (3, 5) and (-1, 1)	A1,A1	A1 for each 'pair' from a correct quadratic equation, correctly obtained.
	Midpoint (1, 3)	B1cao	midpoint
	(Gradient - 1) Perpendicular bisector $y = 4 - x$	M1	perpendicular bisector, must be using <i>their</i> perpendicular gradient and <i>their</i> midpoint
	Meets the curve again if $x^2 + 10x - 15 = 0$ or $y^2 - 18y + 41 = 0$	M1	substitution and simplification to obtain a three term quadratic equation in one variable.
	leading to $x = -5 \pm 2\sqrt{10}$, $y = 9 \mp 2\sqrt{10}$	A1,A1	A1 for each 'pair'
	$CD^2 = (4\sqrt{10})^2 + (4\sqrt{10})^2$	M1	Pythagoras using <i>their</i> coordinates from solution of second quadratic. $(x_1 - x_2)^2 + (y_1 - y_2)^2$ must be seen if not using correct coordinates.
$CD = 8\sqrt{5}$	A1	A1 for $8\sqrt{5}$ from $\sqrt{320}$ and all correct so far.	

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12 (a)	$2^{2x-1} \times 2^{2(x+y)} = 2^7$ and $\frac{3^{2(2y-x)}}{3^{3(y-4)}} = 1$ $2x - 1 + 2(x + y) = 7$ oe $2(2y - x) = 3(y - 4)$ oe leading to $x = 4, y = -4$ <u>Example of Alternative method</u> Method mark as above $2x - 1 + 2(x + y) = 7$ leading to $y = \frac{(8 - 4x)}{2}$ Correctly substituted in $\frac{3^{2(2y-x)}}{3^{3(y-4)}} = 1$ Leading to $2\left(\frac{2(8 - 4x)}{2} - x\right) = 3\left(\frac{(8 - 4x)}{2} - 4\right)$ Leading to $x = 4$ and $y = -4$	M1	expressing 4^{x+y} , 128 as powers of 2 and 9^{2y-x} , 27^{y-4} as powers of 3
		A1 A1 A1	Correct equation from correct working Correct equation from correct working for both
		M1 A1	As before One of the correct equations in x and y
		A1 A1	Correct, unsimplified, equation in x or y only Both answers
(b)	$(2(5^z) - 1)(5^z + 1) = 0$ leading to $2.5^z = 1$ ($5^z = -1$) $5^z = 0.5$	M1	solution of quadratic
		A1	correct solution
		DM1	correct attempt to solve $2.5^z = k$, where k is positive
	$z = \frac{\log 0.5}{\log 5}$ or $z = -0.431$ or better	A1	must have one solution only